AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Mechanics 2B

Monday 25 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working, otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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4	
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6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

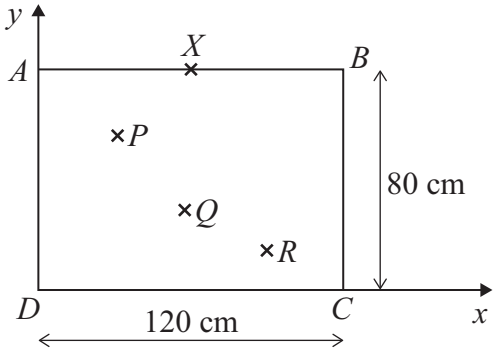
Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A uniform rectangular lamina $ABCD$ has mass $7\,\text{kg}$. It has one side AB of length $120\,\text{cm}$ and another side BC of length $80\,\text{cm}$ and is used as a display board for a small zoo.

Three small animal shapes are positioned on the lamina at the points P, Q and R as shown in the diagram.



Not drawn to scale

Coordinate axes are drawn as shown with the origin at the point D. The three animal shapes are to be treated as particles at the points as below.

Lion of mass $6\,\text{kg}$ at the point P, which has coordinates $(20, 60)$.

Giraffe of mass $3\,\text{kg}$ at the point Q, which has coordinates $(60, 30)$.

Crocodile of mass $4\,\text{kg}$ at the point R, which has coordinates $(80, 20)$.

(a) Find the distance of the centre of mass of the display board with the animals attached

(i) from the line AD.

[3 marks]

(ii) from the line DC.

[2 marks]

(b) The display board is suspended by a rope from a point X which is the midpoint of AB.

It is required that in equilibrium the display board will hang so that AB is horizontal.

To ensure this, a zebra will be fixed to a point on BC.

Find the mass of the zebra.

[4 marks]

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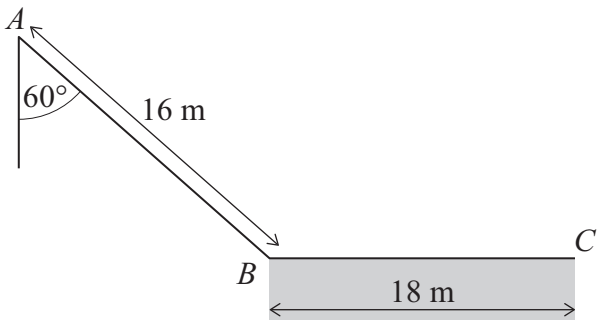
2 A slide in a water park may be modelled as a smooth slope, AB, of length 16 metres inclined at an angle of $60°$ to the vertical. At the bottom of the slope there is a rough horizontal surface, BC, of length 18 metres.



Jay, a child of mass $21\,\text{kg}$, slides down the slope and then along the horizontal surface. At A his velocity is $2\,\text{m s}^{-1}$ and the coefficient of friction between Jay and the rough horizontal surface BC is μ.

Assume that no energy is lost at B and that air resistance is to be ignored.

(a) Calculate the kinetic energy of Jay at A.

[2 marks]

(b) By using conservation of energy:

(i) find the kinetic energy of Jay when he reaches B;

[3 marks]

(ii) find the speed of Jay when he reaches B.

[2 marks]

(c) Jay comes to rest when he reaches C.

Find μ.

[3 marks]

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3 A particle, of mass $2\,\text{kg}$, moves in a horizontal plane under the action of a resultant force, $\mathbf{F}$ newtons.

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively. At time t seconds, the velocity of the particle, $\mathbf{v}\,\text{m s}^{-1}$, is given by

$$\mathbf{v} = (12t - t^3)\mathbf{i} - 6e^{-2t}\mathbf{j} \qquad (t \geqslant 0)$$

(a) Find an expression for the acceleration of the particle at time t.

[2 marks]

(b) (i) Find an expression for $\mathbf{F}$ at time t.

[2 marks]

(ii) Find the magnitude of $\mathbf{F}$ when $t = 0$.

[2 marks]

(c) Find the value of t when $\mathbf{F}$ acts due north.

[2 marks]

(d) When $t = 0$, the particle is at the point with position vector $4\mathbf{i} - 2\mathbf{j}$

Find the position vector, $\mathbf{r}$ metres, of the particle at time t.

[5 marks]

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4 A car, of mass $900\,\text{kg}$, travels around a roundabout on a horizontal, circular path at a constant speed of $12\,\text{m}\,\text{s}^{-1}$. The radius of its circular path is 80 metres. Assume that there is no resistance to the motion of the car and that the car can be modelled as a particle.

(a) A frictional force, directed towards the centre of the car's circular path, acts on the car as it moves. Show that the magnitude of this frictional force is $1620\,\text{N}$.

[2 marks]

(b) The coefficient of friction between the car and the road is μ.

Find the least possible value of μ.

[2 marks]

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5 A car has a maximum power of 32 kilowatts.

When the car is moving at a speed of $v\,\mathrm{m\,s^{-1}}$, a total resistance force of magnitude kv newtons acts on the car where k is a constant.

(a) When the car is moving along a straight horizontal road, the maximum speed of the car is $40\,\mathrm{m\,s^{-1}}$.

Show that $k = 20$

[2 marks]

(b) When the car is travelling at $18\,\mathrm{m\,s^{-1}}$ the car starts to travel down a slope which is inclined at an angle θ to the horizontal, where $\sin\theta = 0.1$

The power exerted by the car's engine is reduced to zero. After the power exerted by the car has been reduced to zero for t seconds, its speed is $v\,\mathrm{m\,s^{-1}}$.

The mass of the car is $600\,\mathrm{kg}$.

(i) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{3g - v}{30}$

[3 marks]

(ii) Find t in terms of v and g.

[5 marks]

(iii) Hence find the time for the speed of the car to increase from $18\,\mathrm{m\,s^{-1}}$ to $22\,\mathrm{m\,s^{-1}}$.

Give your answer to 3 significant figures.

[3 marks]

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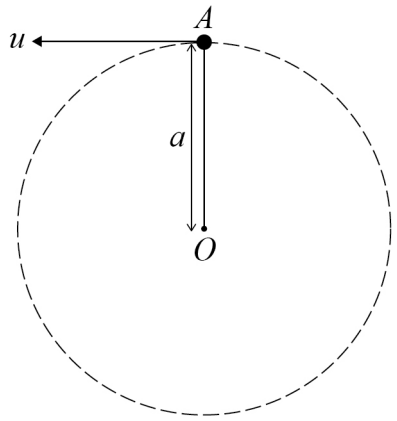
6 A small bead, of mass m, is attached by a light inextensible string, of length a, to a fixed point O.

The string is taut and the bead is held at a point A, where A is vertically above O.

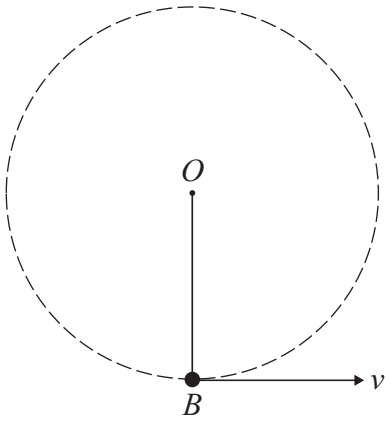
The bead is set into vertical circular motion with an initial horizontal velocity, u, as shown in **Figure 1**. The string does not become slack in the subsequent motion.

Figure 1



The speed of the bead at the point B where B is vertically below O is v, as shown in **Figure 2**.

Figure 2



The ratio of the tensions in the string when the bead is at the two points A and B is $5 : 7$.

(a) Find u in terms of a and g.

[6 marks]

(b) Find the ratio $u : v$

[2 marks]

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7 A particle, P, of mass $8\,\text{kg}$ is resting on a rough horizontal plane.

The particle is attached to one end of a light elastic string AP of natural length 2 metres and modulus of elasticity $160\,\text{N}$. It is also attached to another light elastic string BP of natural length 3 metres and modulus of elasticity $240\,\text{N}$. The points A and B are fixed on the horizontal plane 7 metres apart.

(a) The particle is held at the point C where the length of AC is 2 metres and the length of BC is 5 metres. The points A, B and C are in a straight line.

Show that the total elastic energy of the two strings when the particle is in this position is $160\,\text{J}$.

[2 marks]

(b) The particle is released from rest at point C. The coefficient of friction between the particle and the rough plane is μ. In the subsequent motion, and before the particle first comes to rest, the particle has speed $v\,\text{m s}^{-1}$ when it is x metres from C.

Find v in terms of x, μ and g.

[5 marks]

(c) Find the value of x, in terms of μ and g, when the speed of the particle is a maximum.

[2 marks]

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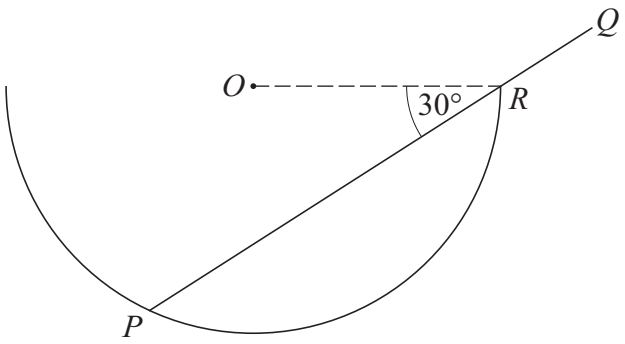
8 A smooth hollow hemisphere, of radius r, is fixed so that its rim is in a horizontal plane, with the centre of the rim being the point O. A smooth uniform rod PQ, of mass m, is in equilibrium, with one end P resting on the inside of the hemisphere and the point R on the rod being in contact with the rim of the hemisphere. The rod, of length l, is inclined at an angle of $30°$ to the horizontal, as shown in the diagram.



Find r in terms of l.

[9 marks]

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